

AMC 8 Cheat Sheet

Every formula, one printable reference · math.ahaanamg.com

Memorize the basics first. Aim for a 15+ score before reaching for the advanced material at the bottom of each section. Trying for 25? See "Reaching for a Perfect Score" on the site for the full climb.

ARITH Arithmetic & Operations

FORMULAS TO KNOW COLD

- **PEMDAS:** Parentheses, Exponents, Mul/Div, Add/Sub.
- **Arithmetic series:** $\text{sum} = (\text{first} + \text{last}) / 2 \times \text{count}$.
- **Sum $1+2+\dots+n = n(n+1)/2$.** ($1+2+\dots+100 = 5050$.)
- **Sum $1+3+5+\dots+(2n-1) = n^2$** (sum of first n odd numbers).
- **Distributive:** $a \times b + a \times c = a \times (b + c)$.
- **Distributing minus over parens:** $a - (b + c) = a - b - c$.
- **Average \leftrightarrow total:** $\text{total} = \text{average} \times \text{count}$.
- **Deviations from the mean sum to zero.**
- **Keep-fractions:** after $p\%$ off \rightarrow multiplier $(1 - p/100)$.
- **Median position:** $(n+1)/2$ if odd; average positions $n/2$ and $n/2+1$ if even.
- **Powers of 10:** $10^3 = 1,000$; $10^6 = \text{million}$; $10^9 = \text{billion}$.

NT Number Theory

FACTS TO KNOW COLD

- **Primes ≤ 50 :** 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47.
- **The only even prime is 2.** 1 is NOT prime.
- **To test if n is prime:** trial-divide by primes $\leq \sqrt{n}$. Beyond that, no factors can hide.
- **Divisibility by 3 or 9:** digit sum.
- **Divisibility by 4:** last 2 digits.
- **Divisibility by 8:** last 3 digits.
- **Divisibility by 11:** alternating digit sum.
- **Divisibility by 7:** chop last digit, double it, subtract from what's left.
- **Divisibility by 13:** chop last digit, multiply by 4, add to what's left.
- **1001 = $7 \cdot 11 \cdot 13$** — split into 3-digit groups, alternating sum tests all three at once.
- **Difference of squares:**
 $a^2 - b^2 = (a - b)(a + b)$.
- **Divisor count formula:** for $n = p^a \cdot q^b \cdot \dots$,
 $\text{count} = (a+1)(b+1)\dots$
- **Perfect-square \leftrightarrow odd number of divisors.**
Every other number has an even count.
- **Perfect-square divisors of n :** count divisors using only EVEN exponents.
- **Divisors pair up:** $d \leftrightarrow n/d$, product = n .
- **GCD(a, b) \times LCM(a, b) = $a \times b$** (two positive integers — fails for three or more).
- **Euclidean algorithm:** $\text{gcd}(a, b) = \text{gcd}(b, a \bmod b)$. Replace the bigger with the leftover until you hit 0.
- **Units-digit cycles:** $2/3/7/8 \rightarrow$ period 4; $4/9 \rightarrow$ period 2; $0/1/5/6 \rightarrow$ stuck.
- **Digit sum $\equiv N \pmod{9}$.**

CONVERSIONS TO MEMORIZE

- $1/2 = 0.5 = 50\%$; $1/3 \approx 0.333 = 33\frac{1}{3}\%$; $2/3 \approx 0.667 = 66\frac{2}{3}\%$
- $1/4 = 0.25 = 25\%$; $3/4 = 0.75 = 75\%$
- $1/5 = 0.2 = 20\%$; $2/5 = 0.4 = 40\%$; $3/5 = 0.6 = 60\%$; $4/5 = 0.8 = 80\%$
- $1/6 \approx 0.167$; $5/6 \approx 0.833$
- $1/8 = 0.125 = 12.5\%$; $3/8 = 0.375$; $5/8 = 0.625$; $7/8 = 0.875$
- $1/9 \approx 0.111$; $1/11 \approx 0.0909$; $1/12 \approx 0.0833$
- **KCF for division:** Keep, Change, Flip — keep the first, change \div to \times , flip the second.
- **Telescoping product:** $(1-1/2)(1-1/3)\dots(1-1/N) = 1/N$.
- **Telescoping sum:** $1/(n(n+1)) = 1/n - 1/(n+1)$.
- **+25% then -20% returns to start** (because $1.25 \times 0.8 = 1$).
- **+1/n undoes with -1/(n+1).** +25% ($=+1/4$) undoes with -20% ($=-1/5$).

RATES Ratios, Rates & Proportions
FORMULAS / FACTS TO KNOW COLD

- **D = S × T** (and the two cousins). Same as **Work = Rate × Time**.
- **Average speed = total distance ÷ total time.** NEVER the average of the speeds (unless times are equal).
- **1 hour = 3600 seconds = 60 minutes.**
- **1 mile = 5280 feet.** 1 km = 1000 m. 1 yard = 3 feet.
- **1 mph ≈ 1.467 ft/s** (so 60 mph = 88 ft/s).
- **Equal-distance round trip avg speed:** $2ab / (a+b)$ (harmonic mean).
- **Equal-time legs avg speed:** $(a+b)/2$ (simple average).
- **Together-time** (A alone takes a, B alone takes b): $T = ab / (a+b)$.
- **Same-direction closing speed:** faster — slower.
- **Opposite-direction closing speed:** sum of speeds.
- **Exponential growth:** $V(n) = V_0 \cdot r^n$. Compound interest: $V_0 \cdot (1 + p/100)^n$.

FORMULAS TO KNOW COLD

- **Linear:** $y = mx + b$. Slope $m = \text{rise/run}$.
- **Arithmetic sequence:** $a_n = a_1 + (n-1)d$.
Sum = $n(a_1 + a_n)/2$.
- **Geometric sequence:** $a_n = a_1 \cdot r^{(n-1)}$.
- **Difference of squares:** $a^2 - b^2 = (a+b)(a-b)$.
(Mental math: $51 \cdot 49 = 50^2 - 1 = 2499$.)
- **Square of binomial:** $(a \pm b)^2 = a^2 \pm 2ab + b^2$.
- **Sum $1+2+\dots+n = n(n+1)/2$.**
- **Sum $1+3+5+\dots+(2n-1) = n^2$.**
- **Sum of squares $1^2+2^2+\dots+n^2 = n(n+1)(2n+1)/6$.**
- **Sum of cubes $1^3+2^3+\dots+n^3 = [n(n+1)/2]^2$.**
- **Sum of n consecutive integers = n × middle.**
- **Average of an arithmetic sequence = (first + last) / 2.**
- **Exponent rules:** $x^a \cdot x^b = x^{a+b}$; $x^a/x^b = x^{a-b}$;
 $(x^a)^b = x^{ab}$; $x^{-a} = 1/x^a$; $x^0 = 1$.
- **Fibonacci:** 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...
(each = sum of previous two).
- **Digit-counts for pages:** 1-9 → 9, 10-99 → 180, 100-999 → 2700.

FORMULAS TO KNOW COLD

- **Rectangle:** $A = lw$, $P = 2(l + w)$.
- **Square:** $A = s^2$, $P = 4s$, diagonal = $s\sqrt{2}$.
- **Triangle:** $A = \frac{1}{2} b h$. Pythagorean: $a^2 + b^2 = c^2$.
- **Equilateral triangle (side s):** $A = (\sqrt{3}/4) s^2$.
Height = $(\sqrt{3}/2) s$.
- **Parallelogram:** $A = b h$ ($h \perp$ base).
- **Trapezoid:** $A = \frac{1}{2} (b_1 + b_2) h$.
- **Regular hexagon (side s):** $A = (3\sqrt{3}/2) s^2$ (= 6 equilateral triangles of side s).
- **Circle:** $A = \pi r^2$, $C = 2\pi r$. Sector: $\pi r^2 \cdot \theta/360$.
Arc: $2\pi r \cdot \theta/360$.
- **Cube:** $SA = 6 s^2$, $V = s^3$. Space diagonal = $s\sqrt{3}$.
- **Box (a × b × c):** $SA = 2(ab + bc + ca)$, $V = abc$. Space diagonal = $\sqrt{a^2 + b^2 + c^2}$.
- **Cylinder:** $V = \pi r^2 h$, $SA = 2\pi r^2 + 2\pi r h$.
- **Cone (radius r, height h):** $V = \frac{1}{3} \pi r^2 h$.
- **Sphere:** $V = (4/3) \pi r^3$, $SA = 4\pi r^2$.
- **Distance between two points:** $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- **Equation of a circle:** $(x - a)^2 + (y - b)^2 = r^2$ (center (a,b), radius r).
- **Pick's Theorem (lattice polygons):** $A = I + B/2 - 1$.
- **13-14-15 triangle:** area = 84 (splits into 5-12-13 + 9-12-15).
- **Pythagorean triples:** 3-4-5, 5-12-13, 8-15-17, 7-24-25, 9-40-41, 20-21-29.
- **Special right triangles:** 45-45-90 sides 1:1: $\sqrt{2}$.
30-60-90 sides 1: $\sqrt{3}$:2.
- **Polygon interior angle sum:** $(n - 2) \cdot 180^\circ$.
- **Regular polygon angles:** interior = $(n - 2) \cdot 180/n$; exterior = $360/n$. Pentagon 108, hexagon 120, octagon 135, n-gon $9 \rightarrow 140$, 10-gon $\rightarrow 144$.
- **Inscribed right triangle:** if a triangle inscribed in a circle has a side equal to the diameter, the opposite angle is 90° .
- **Tangent \perp radius:** a tangent line meets the radius at the touch-point at 90° .

QUADRILATERAL AREAS

- square s^2 · rectangle / parallelogram $b \cdot h$
- **rhombus** $\frac{1}{2} \cdot d_1 \cdot d_2$ (the diagonals — not the sides!)
- trapezoid $\frac{1}{2} (b_1 + b_2) \cdot h$ (average the parallel sides, times height)

FORMULAS TO KNOW COLD

- **Multiplication principle:** independent steps multiply.
- **Factorial:** $n! = n \cdot (n - 1) \cdot \dots \cdot 2 \cdot 1$. $5! = 120$. $6! = 720$. $7! = 5040$. ($0! = 1$ by convention.)
- **Permutations** (order matters): $P(n, k) = n! / (n - k)!$.
- **Combinations** (order doesn't matter): $C(n, k) = n! / (k! \cdot (n - k)!)$.
- **Symmetry:** $C(n, k) = C(n, n - k)$.
- **Handshake formula:** $C(n, 2) = n(n - 1) / 2$.
- **Subsets of an n-element set:** 2^n .
- **Circular arrangements:** $(n - 1)!$ (rotations of the same arrangement count once).
- **Letters with repeats:** $n! / (d_1! \cdot d_2! \cdot \dots)$.
- **Stars and bars:** identical N items among k distinct recipients (zeros allowed) = $C(N + k - 1, k - 1)$.
- **Probability:** favorable / total (with equally-likely outcomes).
- **Independent events:** $P(A \text{ and } B) = P(A) \cdot P(B)$.
- **Mutually exclusive:** $P(A \text{ or } B) = P(A) + P(B)$.
- **Complement:** $P(\text{not } A) = 1 - P(A)$.
- **Inclusion-exclusion:** $|A \cup B| = |A| + |B| - |A \cap B|$.
- **Lattice paths (right/up):** $C(m + n, m)$.
- **Pigeonhole (guarantee N from k):** $k(N - 1) + 1$ items.

LOGIC Logic & Word Problems

FACTS TO KNOW

- **Contrapositive:** 'If A then B' \equiv 'If not B then not A'.
- **Converse** (not equivalent): 'If B then A'.
- **Inverse** (not equivalent): 'If not A then not B'.
- **De Morgan:** $\text{NOT}(A \text{ and } B) = (\text{NOT } A) \text{ or } (\text{NOT } B)$. $\text{NOT}(A \text{ or } B) = (\text{NOT } A) \text{ and } (\text{NOT } B)$.
- **Counterexample disproves a 'for all'.** One specific example with A and not-B kills 'A \rightarrow B'.
- **One example doesn't prove a 'for all'.** You'd need a general argument.
- **For 'there exists' claims:** one example proves; you'd need to check every case to disprove.

THE SEVEN HABITS

1. **Read the question twice.** Underline the question word. What does it actually ask?
2. **Circle the tiny words.** “inclusive,” “except,” “positive,” “non-negative,” “at most,” “different,” “distinct.”
3. **Match the units.** Cents vs dollars. Minutes vs hours. Inches vs feet.
4. **Plug your answer back in.** Does it satisfy every condition the problem stated?
5. **Don't trust mental math past 3 digits.** Write it on scratch.
6. **Re-read your final letter.** The bubble has to match your work.
7. **Two-minute rule.** If a problem's taking too long, mark it and move on.

USE THE ANSWERS

The answer choices are part of the problem. Ask: what do they have in common? What's DIFFERENT about each?

- All whole numbers → the answer is a whole number.
- Three choices obviously too big or too small → cross them off without solving.
- Choices differ by units digit → find the units digit; you're done.
- Choices differ by parity → find the parity first.